# Internet Appendix:
## A Simple Model of Pricing Error at Earnings Announcements Under Asymmetric Timeliness

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**Abstract**

This note presents a simple algebraic model formalizing the mechanism in Barth, Landsman, Raval, and Wang (2020). When earnings exhibits asymmetric timeliness (ATC), investors must disaggregate announced earnings into good-news and bad-news components to assess value. The model shows that pricing error from imperfect disaggregation is increasing in ATC. Because investors can reduce this error by spending more time analyzing the announcement, the model predicts that higher ATC is associated with slower resolution of investor disagreement and uncertainty—the central empirical finding of the paper.

## 1 Setup

Consider a firm whose economic value changes by $V$ during the period, where

$$V = G - B, \tag{1}$$

with $G \geq 0$ representing the aggregate magnitude of good-news events (gains in economic value) and $B \geq 0$ representing the aggregate magnitude of bad-news events (losses in economic value).

Not all of $G$ and $B$ is recognized in earnings in the same period. Let $\alpha_G \in (0, 1]$ and $\alpha_B \in (0, 1]$ denote the proportions of good and bad news, respectively, that are recognized in earnings. Announced earnings is then

$$X = \alpha_G\, G - \alpha_B\, B. \tag{2}$$

Define the *recognized* components of earnings as

$$X_G \equiv \alpha_G\, G \geq 0, \qquad X_B \equiv \alpha_B\, B \geq 0, \tag{3}$$

*This note presents a toy model that was developed as part of the research underlying Barth, Landsman, Raval, and Wang (2020, *The Accounting Review*). The model was not included in the published paper. It is posted here as a companion piece to formalize the economic mechanism.

so that $X = X_G - X_B$. Define the corresponding earnings multiples (the inverse of recognition proportions) as

$$\beta_G \equiv \frac{1}{\alpha_G}, \qquad \beta_B \equiv \frac{1}{\alpha_B}. \tag{4}$$

Then true value can be written as a function of the recognized earnings components:

$$V = \beta_G X_G - \beta_B X_B. \tag{5}$$

# 2 The Symmetric Benchmark

Under *symmetric* timeliness, $\alpha_G = \alpha_B = \alpha$, so $\beta_G = \beta_B = \beta = 1/\alpha$. Earnings reduces to

$$X = \alpha(G - B) = \alpha V,$$

and an investor who observes $X$ can immediately compute

$$V = \beta X = \frac{X}{\alpha}.$$

Because $\alpha$ (equivalently $\beta$) is known prior to the earnings announcement, the investor can calculate $V$ from $X$ without error and without any need to decompose $X$ into its good-news and bad-news components. Information resolution is immediate.

# 3 Asymmetric Timeliness

**Definition 1** (Asymmetric Timeliness)**.** Earnings exhibits *asymmetric timeliness* when $\alpha_B > \alpha_G$, i.e., bad news is recognized more timely than good news. Define the degree of asymmetric timeliness as

$$ATC \equiv \alpha_B - \alpha_G > 0. \tag{6}$$

When $ATC > 0$, the earnings multiples differ: $\beta_G > \beta_B$ (since $\alpha_G < \alpha_B$). The gap between them is

$$\beta_G - \beta_B = \frac{1}{\alpha_G} - \frac{1}{\alpha_B} = \frac{\alpha_B - \alpha_G}{\alpha_G \alpha_B} = \frac{ATC}{\alpha_G \alpha_B} > 0. \tag{7}$$

This means a dollar of recognized good-news earnings maps into more economic value than a dollar of recognized bad-news earnings. An investor can no longer simply multiply total earnings $X$ by a single $\beta$ to recover $V$. Instead, computing value from (5) requires knowing $X_G$ and $X_B$ separately.

**Mapping to Basu (1997).** In the Basu regression,

$$X_{i,y} = \beta_0 + \beta_1 DR_{i,y} + \beta_2 R_{i,y} + \beta_3 DR_{i,y} \times R_{i,y} + \nu_{i,y},$$

the coefficient $\beta_2$ captures the timeliness of good news in earnings, and $\beta_2 + \beta_3$ captures the timeliness of bad news. Thus $\alpha_G$ in our model corresponds to $\beta_2$, $\alpha_B$ corresponds to $\beta_2 + \beta_3$, and our $ATC = \alpha_B - \alpha_G$ corresponds to $\beta_3$, the asymmetric timeliness coefficient.

## 4 The Investor's Problem

At the earnings announcement, the investor observes $X$ but not $X_G$ and $X_B$ individually. The investor knows $\beta_G$ and $\beta_B$ (which are properties of the firm's accounting system, observable from prior data), so the challenge is to decompose $X$ into its components.

**Assumption 1** (Disaggregation)**.** The investor holds a prior belief $\theta \in [0, 1)$ about the proportion of bad news in total recognized earnings, defined as

$$\theta \equiv \frac{X_B}{X_G + X_B}.$$

Using $\theta$ and observed $X = X_G - X_B$, the investor estimates the recognized components.[1]

Given $X$ and $\theta$, the investor solves

$$\hat{X}_B = \theta(\hat{X}_G + \hat{X}_B), \qquad \hat{X}_G - \hat{X}_B = X,$$

which yields

$$\hat{X}_G = \frac{(1-\theta)\,X}{1-2\theta}, \qquad \hat{X}_B = \frac{\theta\,X}{1-2\theta}. \tag{8}$$

The investor's estimated value is

$$\hat{V} = \beta_G\,\hat{X}_G - \beta_B\,\hat{X}_B. \tag{9}$$

**Definition 2** (Disaggregation Error and Pricing Error)**.** Let $\theta^*$ denote the true proportion, and define the *disaggregation error* as

$$\nu \equiv X_B - \hat{X}_B = X_G - \hat{X}_G.$$

The equality $X_B - \hat{X}_B = X_G - \hat{X}_G$ follows because both the true and estimated components satisfy $X_G - X_B = X$. The *pricing error* is defined as

$$\varepsilon \equiv V - \hat{V}. \tag{10}$$

Substituting (5) and (9):

$$\begin{aligned}\varepsilon &= \big(\beta_G\,X_G - \beta_B\,X_B\big) - \big(\beta_G\,\hat{X}_G - \beta_B\,\hat{X}_B\big)\\ &= \beta_G\big(X_G - \hat{X}_G\big) - \beta_B\big(X_B - \hat{X}_B\big)\\ &= \beta_G\,\nu - \beta_B\,\nu\\ &= (\beta_G - \beta_B)\,\nu. \end{aligned} \tag{11}$$

The unsigned pricing error is

$$|\varepsilon| = (\beta_G - \beta_B)\,|\nu| = \frac{ATC}{\alpha_G\,\alpha_B}\,|\nu|, \tag{12}$$

where the last equality uses (7).

This expression is central to the model. Pricing error decomposes into two factors: (i) the gap between the earnings multiples, which is determined by asymmetric timeliness, and (ii) the magnitude of the disaggregation error.

[1] This requires $\theta < \frac{1}{2}$ when $X > 0$ and $\theta > \frac{1}{2}$ when $X < 0$, so that the system of equations has a feasible solution. When this condition fails (e.g., strongly positive earnings dominated by bad news recognition), the investor cannot coherently decompose $X$ using $\theta$ alone and would need to employ alternative methods, such as the naïve single-beta approach discussed in Appendix A.

# 5 Main Result

**Proposition 1** (Pricing Error is Increasing in ATC)**.** *Holding $\alpha_G$ and $|\nu|$ fixed, the unsigned pricing error $|\varepsilon|$ is strictly increasing in ATC whenever $|\nu| > 0$:*

$$\frac{\partial\,|\varepsilon|}{\partial\,ATC} = \frac{|\nu|}{(\alpha_G + ATC)^2} > 0 \quad \textit{for} \;\; |\nu| \neq 0.$$

*Proof.* Since $\alpha_B = \alpha_G + ATC$, we can write

$$|\varepsilon| = \left(\frac{1}{\alpha_G} - \frac{1}{\alpha_G + ATC}\right)|\nu|.$$

Differentiating with respect to $ATC$:

$$\frac{\partial\,|\varepsilon|}{\partial\,ATC} = \frac{|\nu|}{(\alpha_G + ATC)^2} = \frac{|\nu|}{\alpha_B^2} > 0$$

whenever $|\nu| > 0$. □

**Corollary 1** (Perfect Information)**.** *When $\nu = 0$ (i.e., the investor knows the true decomposition), $\varepsilon = 0$ regardless of ATC.*

The intuition is straightforward: asymmetric timeliness drives a wedge between the earnings multiples for good and bad news. Any error in decomposing earnings is amplified by this wedge. The larger the wedge (higher ATC), the more costly the decomposition error.

# 6 Implications

## 6.1 Slower Resolution of Investor Uncertainty

Proposition 1 is a static result: at any point in time, higher ATC produces larger pricing error for a given disaggregation error. To connect this to the *speed* of resolution documented in Barth et al. (2020), suppose the disaggregation error decreases over time as investors process information:

$$|\nu(t)| \text{ is decreasing in } t,$$

where $t$ denotes time elapsed since the earnings announcement. Then the pricing error at time $t$ is

$$|\varepsilon(t)| = \frac{ATC}{\alpha_G\,\alpha_B}\,|\nu(t)|.$$

Suppose investors are willing to trade once their pricing error falls below some threshold $\tau > 0$. The time $t^*$ at which $|\varepsilon(t^*)| = \tau$ satisfies

$$|\nu(t^*)| = \frac{\tau\,\alpha_G\,\alpha_B}{ATC}.$$

Because $|\nu(\cdot)|$ is decreasing, a higher ATC requires $|\nu|$ to fall to a lower level before the threshold is met, which takes more time. Therefore, $t^*$ is increasing in ATC: *investors take longer to begin trading when asymmetric timeliness is greater.* This predicts a lower proportion of trading volume and volatility in the initial announcement period relative to the full announcement period—precisely the negative relation between ATC and $EA\_VOLM$ and $EA\_VOLA$ documented in Barth et al. (2020).

### 6.2 Investor Disagreement (Trading Volume)

The model above treats investors as having a common $\theta$, which generates uncertainty but not disagreement. To generate disagreement (and hence trading volume), suppose investors are heterogeneous in their priors:

Investor $j$ holds belief $\theta_j$, producing estimated value $\hat{V}_j = \beta_G \hat{X}_{G,j} - \beta_B \hat{X}_{B,j}$.

Two investors $j$ and $k$ with $\theta_j \neq \theta_k$ will disagree about firm value. The magnitude of their disagreement is

$$|\hat{V}_j - \hat{V}_k| = (\beta_G - \beta_B)\,|\nu_j - \nu_k| = \frac{ATC}{\alpha_G\,\alpha_B}\,|\nu_j - \nu_k|.$$

The same ATC-driven wedge that amplifies individual pricing error also amplifies cross-investor disagreement. Higher ATC causes the same dispersion in $\theta$ beliefs to produce larger valuation differences, generating more trading volume. Moreover, because disaggregation takes time, this elevated disagreement persists longer for firms with higher ATC, predicting a negative relation between ATC and $EA\_VOLM$.

### 6.3 Insider Trading

Firm insiders know the true decomposition of earnings ($\theta^* = \theta$), so $\nu = 0$ and $\varepsilon = 0$ regardless of ATC. Outside investors, facing $|\varepsilon| > 0$, demand compensation for pricing error risk, which temporarily elevates the discount rate and depresses the stock price below its fundamental value. Insiders, who are not subject to this pricing error, can exploit the temporary mispricing by purchasing shares during the remaining announcement period. This predicts that net insider purchases during days $(3, 20)$ are positively associated with ATC, consistent with the portfolio-level evidence in Barth et al. (2020).

## 7 Numerical Example

To isolate the effect of ATC on pricing error, suppose the investor's disaggregation error is $|\nu| = 1$ regardless of the level of ATC. Fix $\alpha_B = 1$ (bad news is fully timely) and vary $\alpha_G$:

| | $\alpha_G$ | $\alpha_B$ | $ATC$ | $\beta_G$ | $\beta_B$ | $\beta_G - \beta_B$ | $\lvert\varepsilon\rvert$ |
|---|---|---|---|---|---|---|---|
| Symmetric | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 | 0.00 | 0.00 |
| Low ATC | 0.80 | 1.00 | 0.20 | 1.25 | 1.00 | 0.25 | 0.25 |
| Med. ATC | 0.50 | 1.00 | 0.50 | 2.00 | 1.00 | 1.00 | 1.00 |
| High ATC | 0.25 | 1.00 | 0.75 | 4.00 | 1.00 | 3.00 | 3.00 |

Table 1: Pricing error for a fixed disaggregation error of $|\nu| = 1$. As ATC increases, the gap $\beta_G - \beta_B$ widens, and the same disaggregation error produces a larger pricing error. Under symmetric timeliness ($ATC = 0$), pricing error is zero regardless of $\nu$.

The table shows that the same unit of disaggregation error produces pricing error of \$0.25 in the low-ATC case but \$3.00 in the high-ATC case—a 12-fold increase. The symmetric case makes the point starkly: when $ATC = 0$, the earnings multiples are identical, and the investor can compute value from a single $\beta$ with no need to decompose at all.

**Concrete example.** Suppose $G = 10$, $B = 4$, so $V = 6$. Under medium ATC ($\alpha_G = 0.50$, $\alpha_B = 1.00$): recognized earnings is $X = 0.5 \times 10 - 1.0 \times 4 = 1$, with true recognized components $X_G = 5$ and $X_B = 4$. The true decomposition proportion is $\theta^* = 4/9 \approx 0.444$. Suppose the investor believes $\theta = 0.350$ (i.e., slightly underestimates the proportion of bad news). Then:

$$\hat{X}_G = \frac{0.65 \times 1}{0.30} = 2.167, \quad \hat{X}_B = \frac{0.35 \times 1}{0.30} = 1.167, \quad \nu = 4 - 1.167 = 2.833.$$

The investor's estimated value is $\hat{V} = 2.00 \times 2.167 - 1.00 \times 1.167 = 3.17$, while true value is $V = 6$, yielding $|\varepsilon| = 2.83$. The same investor beliefs under low ATC ($\alpha_G = 0.80$) would produce a much smaller pricing error, because the wedge $\beta_G - \beta_B$ is narrower.

# 8 Discussion

Several features of the model merit discussion.

**Source of the result.** The key driver is the "kink" in the earnings–value mapping created by asymmetric timeliness, not timeliness per se. If $\alpha_G$ and $\alpha_B$ both decrease by the same amount (reducing timeliness symmetrically), $\beta_G - \beta_B$ does not change and $|\varepsilon|$ is unaffected. It is the *differential* in timeliness—the asymmetry—that generates the pricing-error wedge.

**Disaggregation error and ATC.** One might ask whether $\nu$ is itself a function of ATC, which would complicate the comparative static in Proposition 1. In general, $\nu$ depends on the investor's prior $\theta$ and the true $\theta^*$, and $\theta^*$ depends on the relative magnitudes of $\alpha_G G$ and $\alpha_B B$, which involve $\alpha_B$. However, $\nu$ can vary independently of ATC: for fixed ATC, different realizations of $G$ and $B$ produce different $\theta^*$ and hence different $\nu$ for a given investor prior $\theta$. The proposition holds for any fixed $|\nu| > 0$.

**The naïve-beta special case.** A simpler version of the model (described in Appendix A) assumes investors do not attempt to decompose earnings at all, instead applying a single naïve beta $\tilde{\beta}$ to total earnings. This is a special case: it corresponds to an investor who sets $\hat{V} = \tilde{\beta}X$ without distinguishing $X_G$ and $X_B$. Pricing error is also increasing in ATC under this approach. However, the disaggregation framework better matches the mechanism described in the published paper, which emphasizes the complexity of decomposition.

**Condition on $\theta$.** The disaggregation requires $\theta < \frac{1}{2}$ when $X > 0$ and $\theta > \frac{1}{2}$ when $X < 0$. This ensures that the investor's belief about the relative magnitude of bad news is directionally consistent with the sign of announced earnings. This is empirically reasonable:

when earnings is positive, good news typically dominates, and investors will generally believe $\theta < \frac{1}{2}$. When this condition fails, the investor cannot coherently decompose $X$ using $\theta$ alone and would likely revert to a naïve-beta approach, under which Proposition 1 still holds (see Appendix A).

# A Naïve-Beta Approach

Suppose instead of decomposing earnings, the investor applies a single naïve beta to total earnings. Let

$$\tilde{\beta} = w\,\beta_B + (1-w)\,\beta_G, \quad w \in [0,1],$$

be any weighted average of the two multiples. The investor estimates $\hat{V} = \tilde{\beta}\,X$. The pricing error is

$$\begin{aligned} \varepsilon &= V - \tilde{\beta}\,X \\ &= (\beta_G\,X_G - \beta_B\,X_B) - \big[w\,\beta_B + (1-w)\,\beta_G\big](X_G - X_B). \end{aligned} \tag{13}$$

After simplification (collecting terms in $X_G$ and $X_B$), the unsigned error is

$$|\varepsilon| = (\beta_G - \beta_B)\,|w\,X_G - (1-w)\,X_B|.$$

Since $\beta_G - \beta_B = ATC/(\alpha_G \alpha_B)$, the pricing error is increasing in ATC for any fixed $w$ and any $(X_G, X_B)$ such that $wX_G \neq (1-w)X_B$. This generalizes the result to any naïve single-beta rule.

The special case $w = \frac{1}{2}$ (simple average) gives $\tilde{\beta} = \frac{1}{2}(\beta_G + \beta_B)$ and

$$|\varepsilon| = \frac{1}{2}(\beta_G - \beta_B)|X_G - X_B| = \frac{ATC}{2\,\alpha_G\,\alpha_B}\,|X_G - X_B|.$$

# References


Barth, M. E., W. R. Landsman, V. Raval, and S. Wang. Asymmetric timeliness and the resolution of investor disagreement and uncertainty at earnings announcements. *The Accounting Review*, 95(4):23–50, 2020.

Basu, S. The conservatism principle and the asymmetric timeliness of earnings. *Journal of Accounting and Economics*, 24(1):3–37, 1997.